Mathematics teaching:
is the future syncretic?

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Syncretic?

- of or relating to the tendency to combine the characteristic teachings, beliefs or practices of differing systems or philosophy

For example, Gnosticism was a dualistic system that incorporated elements from the Oriental mystery religions, Judaism, and Christianity.
I. INTRODUCTION

II. Maths Wars of Myths
   Traditionalist Myths
   Constructivist Myths

III. Realities of to-day’s undergraduate classroom

IV. Non-IT remedies for to-day’s undergraduate classroom

V. IT remedies for to-day’s undergraduate classroom

VI. CONCLUSIONS
I. INTRODUCTION

Theories of Learning

**Traditionalism**
the world is ...structured, ...
Its structure can be modelled for the learner...thought processes ... are analyzable and decomposable - Johnsson91

**Constructivism**
learners construct meaning from current knowledge structures
Piaget –
Discovery Learning
I. INTRODUCTION

Theories of Learning

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Piaget – Discovery Learning

Guided Teaching
(Scaffolding)
Vygotsky
II. Maths Wars: Traditionalist Myths

intelligence = implicit memory (no conscious awareness) + basic pattern recognition

Cicero tells the story of how Simonides of Ceos was rebuked by Scopas, his patron, for devoting too much space to praising Castor and Pollux in an ode celebrating Scopas' victory in a chariot race. Shortly afterwards, Simonides was told that two young men wished to speak to him; after he had left the banqueting room, the roof fell in.

During the excavation of the rubble, Simonides was called upon to identify the guests. He correlated their identities to their positions at the table before his departure. He drew on this experience to develop a mnemonics system called the 'memory theatre' or 'memory palace'.

Simonides of Ceos
chariot race
memory theatre
memory palace
II. Maths Wars: Traditionalist Myths

Traditionalist myths following from the “memory fallacy” are

- mathematics should be taught using poetry, other mnemonics and coercion
- there is no royal road to mathematics (learning times tables alone can take up to a year)

A Vedic chiti – a sacrificial altar
Math, Math Education, Math Culture

Victor Guskov: Should students know their times tables immaculately?

Jonathan Crabtree: … the Chinese … answered this question two thousand years ago… to master … arithmetic, a student first had to learn the 'Nine nines song'.

http://www.youtube.com/watch?v=7Tg0oH3Ga5g (38 s)

Victor Guskov: And what did they do when a student wouldn't learn the song?

Jonathan Crabtree: Spare the (counting) rod, spoil the child!
Constructivism is an understandable reaction to traditionalism
MYTH 1: Only what students discover for themselves is truly learned

In recent large-scale studies, in some cases, school achievement improved from the 16th percentile to above the 90th percentile (Rebar, 2007). John Hattie's Visible Learning: A Synthesis of Over 800 Meta-Analyses Relating to Achievement (2009) summarizes the results of four meta-analyses that examined Direct Instruction. These analyses incorporated 304 studies of over 42,000 students. Across all of these students, the average effect size was .59 and was significantly larger than those of any other curriculum Hattie studied.

Child-centred, cognitively focused, and open classroom approaches tend to perform poorly on all measures of academic progress.
II. Maths Wars: Constructivist Myths

**MYTH 2:** There are two separate and distinct ways to teach mathematics, … conceptual understanding through a problem solving approach and through drill and kill

Basic Skills vs Conceptual Understanding; a Bogus Dichotomy; Hung-Hsi Wu, Department of Mathematics, University of California, Berkeley: Conceptual advances are invariably built on the bedrock of technique.

**MYTH 3:** Math concepts are best understood and mastered when presented "in context"; in that way, the underlying math concept will follow automatically, even though when story problems take centre stage, the math it leads to is often not practiced or applied widely enough for students to learn how to apply the concept to other problems. The Mathematician and Mathematics Education Reform; Hung-Hsi Wu, University of California, Berkeley
II. Math Wars: Consequences

A form of mathematics stripped of much of its intellectual content has obvious repercussions for higher education and the American economy. Hung-Hsi Wu, a Berkeley mathematician, expressed the view of many of his peers when he wrote in 1997 that the brand of mathematics purveyed by the NCTM’s 1989 report “has the potential to change completely the undergraduate mathematics curriculum and to throttle the normal process of producing a competent corps of scientists, engineers, and mathematicians.” And Larry Faulkner, the past president of the University of Texas in Austin, among others, warned that if national policy doesn’t ensure the development of a large domestic workforce with first-rate technical skills, there is a risk “technological surprise to … economic viability and to the foundations of … security.”

The math wars, which started in debates about pedagogy, may end in questions about the long-term prospects for our society.
### III. Reality of to-day’s undergraduate classroom

<table>
<thead>
<tr>
<th><strong>Student has</strong></th>
<th><strong>Teacher expects</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>limited maths background, ...</td>
<td>ability to use traditional lectures/tutorials</td>
</tr>
<tr>
<td>limited memory</td>
<td>memorise numerous disjointed facts</td>
</tr>
<tr>
<td>limited proficiencies in inquiry at levels of comprehension that require explanatory reasoning</td>
<td>ability to discover explanations (meanings) <em>all by themselves!</em></td>
</tr>
</tbody>
</table>
III. Reality of to-day’s undergraduate classroom

There is a gap between mutual expectations of teacher and Learner in a modern University, particularly when the teacher has a sound maths background and the learner is there just to pick up a few skills without understanding why.

To effect deep learning both teachers and students need to **learn to unlearn** most of their beliefs about each other.
### IV. Non-IT remedies for to-day’s undergraduate classroom

<table>
<thead>
<tr>
<th><strong>Student has</strong></th>
<th><strong>Teacher</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>limited maths background,</td>
<td>guides through all material</td>
</tr>
<tr>
<td>limited memory</td>
<td>restricts the jargon</td>
</tr>
<tr>
<td>limited proficiencies in explanatory reasoning</td>
<td>teaches self-explanations</td>
</tr>
<tr>
<td>test addiction</td>
<td>cures test craving</td>
</tr>
</tbody>
</table>

**How?**
III. Non-IT remedies for today’s undergraduate classroom

**Socratic Dialogue**
- Pedagogical technique best suited to modern times
- It has been used to revolutionise US undergraduate physics teaching

**Eulerian Sequencing**
- A systematic approach to maths as language to analyse (sequence) expressions & find the relevant solution algorithm (sequence of steps) - Euler
- Eulerian sequencing teaches learners to generate self-explanations
III. Non-IT remedies for to-day’s undergraduate classroom

**Socratic dialogue** is important because it gives feedback to both parties.

**Eulerian sequencing** is important, because “The amount learned ... is proportional to the number of self-explanations that a student generates”, that is, comments on a solution step “that contain... domain-relevant information over and above what was stated in the description of the step”
III. Non-IT remedies for to-day’s undergraduate classroom

Evidence of success

1. As a practical outcome, learners acquire ability to “recognise familiar patterns in unfamiliar pictures”.
2. They achieve wider and deeper learning and do not have to be “trained to exams”.
3. Their pass rate increases from 30% to 70% even when offered exam questions they have never seen before.
IV. IT Remedies: Online lectures?

Is the "flipped classroom" – i.e., using online lectures as preparation work for in-person interactions at multiple locations – a viable approach? Can technology “humanise” classroom?

http://www.youtube.com/watch?v=gM95HHl4gLk
IV. IT Remedies: Online lectures by Khan?

Pros
1. Students learn new material at home at their own pace
2. Students interact with teachers and other students solving problems in class
3. You expect mastery (no automatic progression)
4. Time is freed for experimentation

Cons
1. What if students don’t understand the lectures at all?
2. Learning requires “me” time too
3. and 4. Do no need IT for that

Who prepares lectures?
IV. IT Remedies: Online lectures by academics?

2012 Gary S. May, Dean of the College of Engineering at the Georgia Institute of Technology: “MOOCs.. could potentially serve as laboratories to conduct experiments that might reinvent education.”

The first sets of MOOCs data:
2013, The University of Edinburgh, Harvard, MIT:
33 % of respondents were between 25-35 years old and were mostly in the “teaching and education” field or students at university. 70% reported having completed an academic degree (a larger percentage than organisers expected)
Average course certification rates are …6% among all registrants in the course (841,687 registrants with 597,692 unique users)...

It is not clear whether any certificates of attainment have been secured by those who had no prior degree.
IV. IT remedies: MOOCs?

Global one-world classroom? Stanford Professor Thrun, one of creators of Udacity suggests that “in 50 years there will be only 10 institutions in the world delivering higher education”


Advanced
Imprecise language
How and not why
  Monster generator approach
Random monster generator approach to calculus: function
Random monster generator approach to calculus: limit
Random monster generator approach to calculus: indeterminate form
Random monster generator approach to calculus: antiderivative
Despite Professor Thrun’s predictions, while some Universities still intend to use existing Coursera materials developed by faculties at elite universities, others begun to say that they would expect that their own faculties will develop materials for the Coursera platform, making them available at campuses system wide and beyond. The company says that Faculty members will be able to customize existing courses, adding their own lessons and refinements.

How can student learning be optimized in an online environment, and what is the best role of the faculty member in such an environment?

This depends on the course, student sophistication, teacher interests and time available for development of the course.
Back to school…

… teachers … just go cover-to-cover, page-by-page, with blind faith that if someone tells them the book is "aligned to the Common Core" then orchestrating a "Bataan Death March" through exercises and trivial content will produce magical results on state tests.

The same tactic would be applied with MOOCs.
IV. IT Remedies: MOOCs?

STANDARDS, TEXTBOOKS, AND PRACTICE MISALIGNED

When it comes to teaching key math domains, researchers say that teachers tend to follow their textbooks, which are not well-aligned with new Common Core State Standards. In 4th grade, for example, experts suggested that in order to adhere to the standards, twice as much time should be spent teaching fractions than called for in several popular textbook series, resulting in teachers spending half the time experts recommend on this key topic.

Experts suggest, based on Common Core State Standards
Teachers actually do
Textbooks suggest

PERCENT OF TIME SPENT DURING 4TH GRADE

SOURCE: Center for the Study of Curriculum, Michigan State University. Research in progress.
IV. IT Remedies: MOOCs?

Schmidt's last study showed that a quality teacher is a top consideration, followed closely by quality materials that they can use to enhance student understanding.

Well-crafted eBooks and other MOOCs with easy-to-edit/improve content can provide a flexible platform for exploring critical focus in depth - a point that most of these "rush to market" programs are trying to cover up with their alignments.
So, are MOOCs just another pi in the sky?
V. Cognitive tutors

- **A Cognitive Tutor** is an educational software containing an AI component and as such is an example of an expert system.

- **Cognitive Tutors** can trace their roots to the ACT-R (Adaptive Control of Thought—Rational) theory of cognition: cognitive skills are realized by **production rules**. ACT-R model separates memory into two types: **declarative**, which correspond to facts (Washington, D.C. is the capital of US, France is a country in Europe or 2+3=5) and **procedural**, which corresponds to the cognitive skills that operate over facts (knowledge about how to type the letter "Q" on a keyboard, drive, perform addition of large numbers.)

In proposing the model, **John Anderson** presented evidence that by modelling procedural knowledge as production rules we can directly observe student learning; the more opportunities students have to use a given production rule the faster and more accurately they can apply it.
V. Cognitive tutors

Cognitive maths tutors for the middle school have been under development CIRCLE, later sold to Carnegie learning. In 2004 U.S. Department of Education that out of over 800 submissions CIRCLE was one of only two studies that addressed teaching and examining middle school math using a strong research design and showing positive results.

AutoTutor simulates the dialog moves of human tutors while participating in conversations with students. It is currently designed to help college students learn about topics in computer literacy and conceptual physics.

Both groups combine the cognitive psychology and AI.
AutoTutor-1: When a car without headrests on the seats is struck from behind, the passengers often suffer neck injuries. Explain why.

Student-2: I don't know.

AutoTutor-3: Could you at least try to give me an answer? That way, we'll have something as a starting place to work from. Let's try this together. When the collision provides a large force to accelerate the head, what could happen to the neck?

Student-4: What is a force?
AutoTutor-5: I'll take a crack at answering your question. Force is any influence that tends to accelerate an object; a push or pull; measured in newtons. Force is a vector quantity.

Was that what you were looking for?

....

AutoTutor-20: See if you know the answer to this: When the head and body are moving at the same acceleration, they are moving at the same_______?

Student-21: distance

AutoTutor-22: Moving at the same rate.
Ms Lindquist - Cognitive Algebra Tutor for school pupils - can be downloaded from internet.
**e-PACT**

<table>
<thead>
<tr>
<th>DIALOGUE WINDOW</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>STUDENT:</strong> Please generate an exercise.</td>
</tr>
<tr>
<td><strong>TUTOR:</strong> Find ( \frac{d}{dp} e^p ).</td>
</tr>
<tr>
<td><strong>STUDENT:</strong> ( \frac{d}{dp} e^p = p e )</td>
</tr>
<tr>
<td><strong>TUTOR:</strong> Your input is incorrect.</td>
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<td><strong>STUDENT:</strong> ( \frac{d}{dp} e^p = p e )</td>
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<tr>
<td><strong>TUTOR:</strong> The expression on the left of the equal sign should be the same as the expression on the right, only written in another form.</td>
</tr>
<tr>
<td><strong>STUDENT:</strong> ( \frac{d}{dp} e^p = p e )</td>
</tr>
<tr>
<td><strong>TUTOR:</strong> The expression on the left of the equal sign should be the same as the expression on the right, only written in another form. Do not ever put equality sign between the symbol of differentiation and function to differentiate.</td>
</tr>
<tr>
<td><strong>STUDENT:</strong> ( \frac{d}{dp} e^p = p e )</td>
</tr>
</tbody>
</table>
TUTOR: Your input is incorrect.

STUDENT: \[ \frac{d}{dp} e^p = pe \]

TUTOR: There are conventions about the order of maths expressions.

Changing it either changes the meaning of the expression or makes it meaningless.

Similarly, you have to watch when you symbols should be raised or lowered.

In particular, it is customary to write the argument of exponent as a superscript.

STUDENT: \[ \frac{d}{dp} e^p = e^p \]

TUTOR: Your answer is correct!

Please move the cursor to the MENU BAR. Click first on the REQUEST option and then on the NEW EXERCISE.

PROBLEM AND SOLUTION WINDOW

PROBLEM: Find \[ \frac{d}{dp} e^p \].

SOLUTION: \[ \frac{d}{dp} e^p = e^p \]
TUTOR: Find \( \frac{d}{d \cos(u)} \ln(\cos(u)) \).

STUDENT: hint

TUTOR: To understand the solution we use the DIFFERENTIATION SUMMARY icon which contains differentiation TABLE, RULES and DECISION TREE. Start from the top of the DECISION TREE. Your answers will suggest which arrows to follow. What is the first question you have to ask yourself?

STUDENT: What is the differentiation variable and what is the function to differentiate?

TUTOR: OK, the question to ask is

"What is the differentiation variable and what is the function to differentiate?"

So, what is the differentiation variable?

STUDENT: cosu

TUTOR: This answer is correct!

What is the function you are asked to differentiate?
STUDENT: \(-\sin \ln u\)

TUTOR: It seems that you have tried to type the solution to the problem. Am I correct?

STUDENT: \(y\)

TUTOR: You must learn to answer the question you have been asked and not jump ahead.

   Nevertheless, I will accept this attempt.

   Your answer is correct!

Please move the cursor to the MENU BAR. Click first on the REQUEST option and then on the NEW EXERCISE.

PROBLEM AND SOLUTION WINDOW

PROBLEM: Find \(\frac{d}{d \ln(u)} \cos(\ln(u))\).

SOLUTION: \(\frac{d}{d \ln(u)} \cos(\ln(u)) = -\sin(\ln(u))\)
Conclusions: syncretic future

It is possible to deliver semi-traditionalist semi-constructivist rigorous instruction

- requiring much less memorisation than traditional (declarative memory, crystallised intelligence)
- allowing for much more conceptual understanding than traditional and constructivist (procedural memory, fluid intelligence)
- much more mastery than constructivist

This can be achieved through non-IT means and IT means

- careful construction of curriculum
- cooperation between faculty
- guided teaching (syncretic pedagogy)
- MOOCs
- social media
- cognitive tutors (combining all of the above!)